

$$5.5.53 \quad \int \frac{1}{\sqrt{x} e^{-\sqrt{x}}} \sec^2(e^{\sqrt{x}} + 1) dx = ?$$

Solution

We have

$$\int \frac{1}{x^{1/2} e^{-x^{1/2}}} \sec^2(e^{x^{1/2}} + 1) dx$$
$$= \int \boxed{x^{-1/2} e^{x^{1/2}} \sec^2(e^{x^{1/2}} + 1)} dx$$

$$\text{let } u = e^{x^{1/2}} + 1$$

$$du = e^{x^{1/2}} \cdot \left[\frac{1}{2} x^{-1/2} \right] dx$$

$$2 du = x^{-1/2} e^{x^{1/2}} dx$$

$$= \int \sec^2(u) 2 du = 2 \int \sec^2(u) du$$

$$= 2 \tan(u) + C$$

$$= 2 \tan(e^{x^{1/2}} + 1) + C$$

$$= \boxed{2 \tan(e^{\sqrt{x}} + 1) + C} \quad \square$$