

5.5.57

$$\int \frac{dz}{1+e^z} = ?$$

Solution

This one is tricky! We have

$$\int \frac{dz}{1+e^z} \quad \text{let } u = e^z$$

$$du = e^z dz \text{ or } \frac{du}{e^z} = dz$$

$$\text{or } \frac{du}{u} = dz$$

$$= \int \frac{du/u}{1+u} = \int \frac{1}{u(1+u)} du.$$

Notice that $\frac{1}{u} - \frac{1}{1+u} = \frac{(1+u) - (u)}{u(1+u)} = \frac{1}{u(1+u)}$

(this is the really tricky part!). So

$$\int \frac{du}{u(1+u)} = \int \left(\frac{1}{u} - \frac{1}{1+u} \right) du$$

$$= \ln|u| - \ln|1+u| + C$$

$$= \ln|e^z| - \ln|1+e^z| + C$$

$$= \ln(e^z) - \ln(1+e^z) + C$$

$$= \boxed{z - \ln(1+e^z) + C}. \quad \square$$