5.5.73 Solve the initial value problem

$$\frac{ds}{dt} = 12t(3t^2-1)^3 \text{ and } s(1)=3.$$

Solution

Well, $s = s(t) \in \int \left(\frac{ds}{dt}\right) dt$

$$= \int 12t(3t^2-1)^3 \, dt$$

let $u = 3t^2 - 1$

$du = 6t\,dt$

$\frac{du}{6} = t\,dt$

$$= 12 \int u^3 \frac{du}{6} = 2 \int u^3 \, du$$

$$= 2\left(\frac{1}{4}u^4\right) + C = \frac{1}{2}(3t^2-1)^4 + C.$$

So we have $s(t) = \frac{1}{2}(3t^2-1)^4 + k$ for some constant $k$.

Since $s(1) = 3$ is the "initial condition", then we need $s(1) = \frac{1}{2}(3(1)^2-1)^4 + k \equiv 3$

or $\frac{1}{2}(2)^4 + k = 3$ or $8 + k = 3$ or $k = 3 - 8 = -5$.

So, $\boxed{s(t) = \frac{1}{2}(3t^2-1)^4 - 5.}$ $\square$