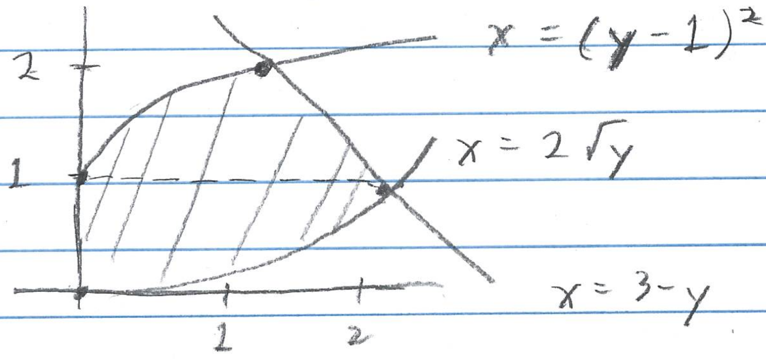
5.6.108 Find the area of the region:



<u>Solution</u>

We are given x as a function of y, so let's integrate with respect to y.
Notice that for the region:

- for $y \in [0,1]$ the function on the left is $x=0$ and the function on the right is $x=2\sqrt{y}$,
- for $y \in [1,2]$ the function on the left is $x=(y-1)^2$ and the function on the right is $x=3-y$.

So, the area is given by TWO integrals:

$$A = \int_0^1 (2\sqrt{y}-0)\,dy + \int_1^2 \left((3-y)-(y-1)^2\right)dy$$

$$= \int_0^1 2y^{1/2}\,dy + \int_1^2 (-y^2+y+2)\,dy$$

$$= 2\left(\tfrac{2}{3}y^{3/2}\right)\Big|_0^1 + \left(\tfrac{-1}{3}y^3+\tfrac{1}{2}y^2+2y\right)\Big|_1^2$$

$$= \left(\frac{4}{3}(1)^{3/2} - \frac{4}{3}(0)^{3/2}\right)$$

$$+\left(\left(\frac{-1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2(2)\right) - \left(\frac{-1}{3}(1)^3 + \frac{1}{2}(1)^2 + 2(1)\right)\right)$$

$$= \frac{4}{3} + \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{-1}{3} + \frac{1}{2} + 2\right)$$

$$= \frac{4}{3} - \frac{8}{3} + 6 + \frac{1}{3} - \frac{1}{2} - 2 = \boxed{3}. \quad \square$$