5.6.33 Evaluate $\int_0^{\pi/2} \tan\left(\frac{x}{2}\right) dx$.

Solution

Notice that $\tan(x/2)$ is continuous on $[0, \pi/2]$, so by the Fundamental Theorem of Calculus, Part 2 we need an antiderivative of $\tan(x/2)$. SO:

$$\int_0^{\pi/2} \tan\left(\frac{x}{2}\right) dx = \int_0^{\pi/2} \boxed{\frac{\sin(x/2)}{\cos(x/2)} dx}$$

let $u = \cos(x/2)$

and $du = -\sin\left(\frac{x}{2}\right)\left[\frac{1}{2}\right] dx$

or $-2\,du = \sin\left(\frac{x}{2}\right) dx$,

ALSO when $x = 0$ then $u = \cos(0/2) = 1$,

when $x = \pi/2$ then $u = \cos(\pi/4) = \sqrt{2}/2$

$$= \int_1^{\sqrt{2}/2} \frac{1}{u}(-2\,du) = -2\int_1^{\sqrt{2}/2} \frac{1}{u}\,du$$

$$= -2\ln|u| \Big|_1^{\sqrt{2}/2} = -2\ln\left(\frac{\sqrt{2}}{2}\right) - (-2\ln(1))$$

$$= -2\ln(\sqrt{2}/2) + 0 = -2\ln(\sqrt{2}/2)$$

$$= -2(\ln\sqrt{2} - \ln 2) = -2\ln(2^{1/2}) + 2\ln(2)$$

$$= -\ln(2) + 2\ln(2) = \boxed{\ln 2}. \quad \square$$