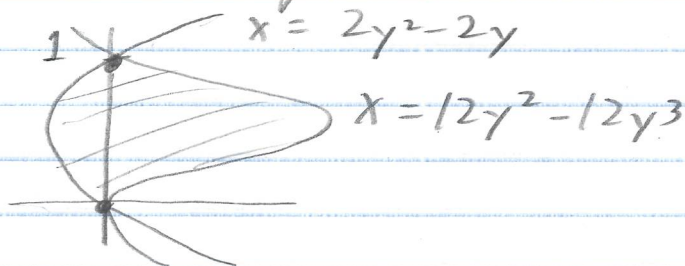


5.6.57 Find the area of the shaded region:



Solution

The area is given by $A = \int_c^d (f(y) - g(y)) dy$

where we have $f(y) = 12y^2 - 12y^3$,
 $g(y) = 2y^2 - 2y$, $c = 0$, and $d = 1$. So

$$A = \int_0^1 ((12y^2 - 12y^3) - (2y^2 - 2y)) dy$$

$$= \int_0^1 (10y^2 - 12y^3 + 2y) dy$$

$$= \left(\frac{10}{3} y^3 - 3y^4 + y^2 \right) \Big|_0^1$$

$$= \left(\frac{10}{3} (1)^3 - 3(1)^4 + (1)^2 \right) - \left(\frac{10}{3} (0)^3 - 3(0)^4 + (0)^2 \right)$$

$$= \frac{10}{3} - 3 + 1 - 0 = \boxed{\frac{4}{3}}. \quad \square$$