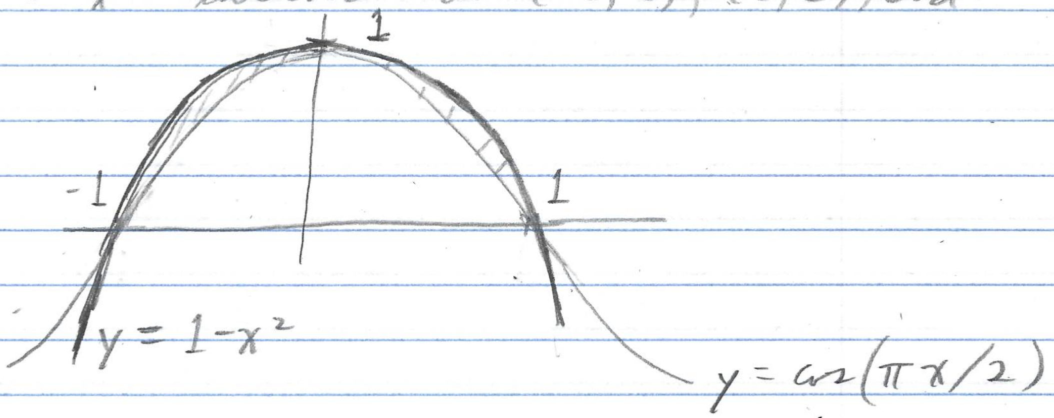
5.6.89 Find the area enclosed by the curves $y = \cos(\pi x/2)$ and $y = 1 - x^2$.

Solution

Notice that the amplitude of $y = \cos(\pi x/2)$ is 1 and the period is $2\pi/(\pi/2) = 4$. From the graph we see that $y = \cos(\pi x/2)$ and $y = 1 - x^2$ intersect at $(-1, 0)$, $(0, 1)$, and $(1, 0)$:



Since $f(x) = 1 - x^2 \geq \cos(\pi x/2)$ for $x \in [-1, 1]$, the area is

$$A = \int_a^b (f(x) - g(x))\,dx = \int_{-1}^{1} \left((1 - x^2) - \cos\left(\frac{\pi x}{2}\right)\right) dx$$

$$= \left(x - \frac{1}{3}x^3 - \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)\right)\Bigg|_{-1}^{1}$$

$$= \left((1) - \frac{1}{3}(1)^3 - \frac{2}{\pi}\sin\left(\frac{\pi(1)}{2}\right)\right)$$

$$- \left((-1) - \frac{1}{3}(-1)^3 - \frac{2}{\pi}\sin\left(\frac{\pi(-1)}{2}\right)\right)$$

$$= \frac{2}{3} - \frac{2}{\pi}(1) - \left(\left(\frac{-2}{3}\right) - \frac{2}{\pi}(-1)\right) = \boxed{\frac{4}{3} - \frac{4}{\pi}}. \quad \square$$