

A.1 #29

Let a be any positive number.

Prove that $|x| > a$ if and only if $x > a$ or $x < -a$.

Proof

Recall that $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$

Suppose that either $x > a$ or $x < -a$.

• If $x > a$ then $x > a > 0$ (since a is positive), and so $|x| = x$ since $x \geq 0$.

Hence, $|x| = x > a$ and $|x| > a$.

• If $x < -a$ then $-x > -(-a) = a > 0$;

so $-(-x) < -(0)$ or $x < 0$ and

hence $|x| = -x$ (since $x < 0$) and

so $|x| = -x > a$. That is, $|x| > a$. \square

Next, suppose $|x| > a$.

• If $x \geq 0$ then $|x| = x$ and so $x = |x| > a$,
or $x > a$.

• If $x < 0$ then $|x| = -x$ and so $-x = |x| > a$
or $-(-x) < -a$ or $x < -a$.

Therefore either $x > a$ or $x < -a$, as claimed. \square