Thomas' Calculus, Early Transcendentals, 12th Edition

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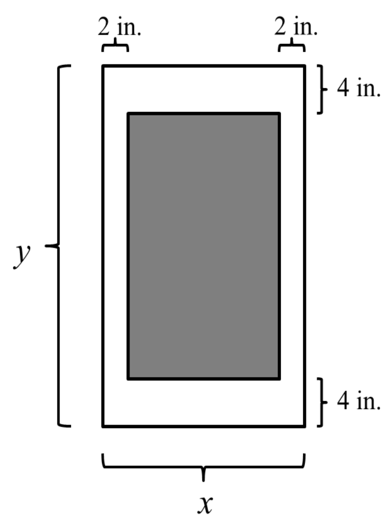
Designing a poster. You are designing a rectangular poster to contain 50 in^2 of printing with a 4-in. margin at the top and bottom and a 2-in. margin at each side. What overall dimensions will minimize the amount of paper used?

Solution. We follow the 5 step method outlined in class.

Step 1. Draw a picture and label the unknowns and constants. We let x represent the width of the poster and y represent with height of the poster. We then have:



Step 2. State the question in terms of the unknowns. The question is to minimize the area A of the poster which, in terms of the unknowns, is $A = xy$.

Step 3. Find a relationship between the unknowns. Due to the margins, the width of the printed area is $x - 4$ in. and the height of the printed area is $y - 8$ in. Since the total printed area is 50 in^2, then $(x-4)(y-8) = 50$. We can solve this for y to get $y - 8 = \dfrac{50}{x-4}$ or

$$y = \frac{50}{x-4} + 8 = \frac{50 + 8(x-4)}{x-4} = \frac{18 + 8x}{x-4}.$$

Step 4. Write the desired quantity as a function of one unknown. The area of the poster in terms of x only is:

$$A = xy = y\left(\frac{18+8x}{x-4}\right) = \frac{18x + 8x^2}{x-4} = A(x).$$

Step 5. Maximize/Minimize the function. Since x cannot equal 4 (if $x = 4$, then there is not printed area), then we need to minimize $A(x)$ for $x \in (4, \infty)$. First, we find the derivative:

$$A'(x) = \frac{[18 + 16x](x - 4) - (18x + 8x^2)[1]}{(x - 4)^2} = \frac{18x + 16x^2 - 72 - 64x - 18x - 8x^2}{(x - 4)^2}$$

$$= \frac{-72 - 64x + 8x^2}{(x - 4)^2} = \frac{8(x^2 - 8x - 9)}{(x - 4)^2} = \frac{8(x - 9)(x + 1)}{(x - 4)^2}.$$

So the derivative is 0 when $x = -1$ and $x = 9$. Since $x = -1$ is not in the interval of interest $(4, \infty)$, then we need not worry about it. Also, the derivative is undefined at $x = 4$, but this is not in the domain of $A(x)$. So the only critical point in the interval $(4, \infty)$ is $x = 4$. We need to see if this corresponds to a maximum or minimum. Let's use the first derivative test. Consider

	$(4, 9)$	$(9, \infty)$
Test Value k	5	10
$A'(k)$	$\frac{8((5) - 9)((5) + 1)}{((5) - 4)^2} = -192$	$\frac{8((10) - 9)((10) + 1)}{((10) - 4)^2} = \frac{88}{36}$
$A'(x)$	$-$	$+$
$A(x)$	DEC	INC

So by the First Derivative Test, $A(x)$ has a minimum at $x = 9$ inches. When the width is $x = 9$ inches, the height is $y = \frac{18 + 8(9)}{(9) - 4} = 18$ inches. So the minimum area is $A = xy = 9 \times 18 = 162$ in^2.