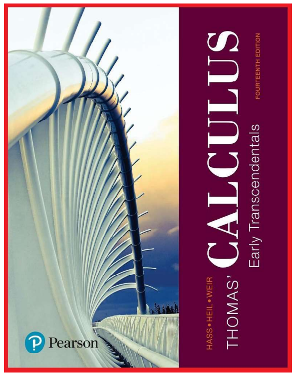
Calculus 1

Chapter 1. Functions

1.5. Exponential Functions—Examples and Proofs

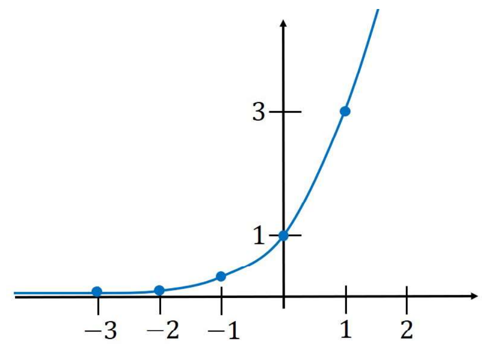


Exercise 1.5.2(a)

Exercise 1.5.2(a). Plot several points and sketch the graph of $y = 3^x$.

Solution. Consider the function values:

x	-3	-2	-1	0	1	2	3
$f(x)$	$3^{-3} = 1/27$	$3^{-2} = 1/9$	$3^{-1} = 1/3$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$

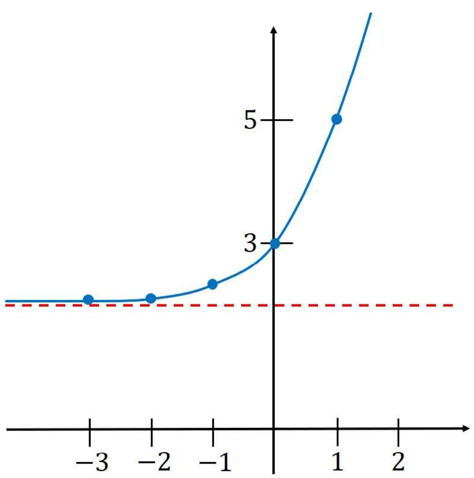


□

Exercise 1.5.8(a)

Exercise 1.5.8(a). Sketch the shifted exponential curve $y = 3^x + 2$.

Solution. We simply shift the graph of $y = 3^x$ from Exercise 1.5.2(a) up by 2 units (because of the "+2"). Notice that the graph of $y = 3^x + 2$ has a horizontal asymptote of $y = 2$.



□

Exercise 1.5.12

Exercise 1.5.12. Use the Law of Exponents (Theorem 1.5.A) to simplify $9^{1/3}9^{1/6}$.

Solution. We have by the Rules for Exponents (Theorem 1.5.A) that $a^x a^y = a^{x+y}$.

So with $a = 9$, $x = 1/3$, and $y = 1/6$, we have $9^{1/3}9^{1/6} = 9^{1/3+1/6} = 9^{1/2} = \boxed{3}$. □

Exercise 1.5.30(a)

Exercise 1.5.30(a). Population Growth.
The population of Silver Run in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year (that is, $k = 0.0275$). **(a)** Estimate the population in 1915 and 1940.

Solution. The population size is given by $y = y_0 e^{kt}$ where $k = 0.0275$, $y_0 = 6250$, and t is time measured in years after 1890.

In 1915, $t = 1915 - 1890 = 25$ and so the population is $y = 6250e^{0.0275(25)} = 6250e^{0.6875} \approx \boxed{12{,}430}$.

In 1940, $t = 1940 - 1890 = 50$ and so the population is $y = 6250e^{0.0275(50)} = 6250e^{1.375} \approx \boxed{24{,}719}$. □

Example 1.5.4

Example 1.5.4. Laboratory experiments indicate that some atoms emit a part of their mass as radiation, with the remainder of the atom reforming to make an atom of some new element. For example, radioactive carbon-14 decays into nitrogen; radium eventually decays into lead. If y_0 is the number of radioactive nuclei present at time zero, the number still present at any later time t will be $y = y_0 e^{-rt}$ where $r > 0$. The number r is the *decay rate* of the radioactive substance. For carbon-14, the decay rate has been determined experimentally to be about $r = 1.2 \times 10^{-4}$ when t is measured in years. Predict the percent of carbon-14 present after 866 years have elapsed.

Solution. With $r = 1.2 \times 10^{-4}$ and $t = 866$, we have $u = y_0 e^{(-0.00012)(866)} = y_0 e^{-0.10392} \approx (0.9013)y_0$. So the percent of carbon-14 present at this time is $(0.9013y_0)/y_0 \times 100\% = \boxed{90.13\%}$.