Exercise 1.5.8(a)

Exercise 1.5.8(a). Sketch the shifted exponential curve \( y = 3^x + 2 \).

Solution. We simply shift the graph of \( y = 3^x \) from Exercise 1.5.2(a) up by 2 units (because of the “+2”). Notice that the graph of \( y = 3^x + 2 \) has a horizontal asymptote of \( y = 2 \).

Exercise 1.5.12

Exercise 1.5.12. Use the Law of Exponents (Theorem 1.5.A) to simplify \( g^{1/3}g^{1/6} \).

Solution. We have by the Rules for Exponents (Theorem 1.5.A) that \( a^x a^y = a^{x+y} \).

So with \( a = 9 \), \( x = 1/3 \), and \( y = 1/6 \), we have \( g^{1/3}g^{1/6} = g^{1/3+1/6} = g^{1/2} = 3 \).
Exercise 1.5.30(a). Population Growth.
The population of Silver Run in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year (that is, $k = 0.0275$). (a) Estimate the population in 1915 and 1940.

**Solution.** The population size is given by $y = y_0 e^{kt}$ where $k = 0.0275$, $y_0 = 6250$, and $t$ is time measured in years after 1890.

In 1915, $t = 1915 - 1890 = 25$ and so the population is $y = 6250 e^{0.0275(25)} = 6250 e^{0.6875} \approx 12,430$.

In 1940, $t = 1940 - 1890 = 50$ and so the population is $y = 6250 e^{0.0275(50)} = 6250 e^{1.375} \approx 24,719$. □

Example 1.5.4.

**Example 1.5.4.** Laboratory experiments indicate that some atoms emit a part of their mass as radiation, with the remainder of the atom reforming to make an atom of some new element. For example, radioactive carbon-14 decays into nitrogen; radium eventually decays into lead. If $y_0$ is the number of radioactive nuclei present at time zero, the number still present at any later time $t$ will be $y = y_0 e^{-rt}$ where $r > 0$. The number $r$ is the decay rate of the radioactive substance. For carbon-14, the decay rate has been determined experimentally to be about $r = 1.2 \times 10^{-4}$ when $t$ is measured in years. Predict the percent of carbon-14 present after 866 years have elapsed.

**Solution.** With $r = 1.2 \times 10^{-4}$ and $t = 866$, we have $u = y_0 e^{-(0.00012)(866)} = y_0 e^{-0.10392} \approx (0.9013)y_0$. So the percent of carbon-14 present at this time is $(0.9013y_0)/y_0 \times 100% = 90.13%$.