Chapter 6. Applications of Definite Integrals

6.6. Work

Recall. When a body moves a distance d along a straight line as a result of being acted on be a force of constant magnitude F in the direction of motion, then the *work* done is $W = Fd$. If we measure forces in pounds and distances in feet, then work is measured in foot-pounds. If we measure forces in newtons and distances in meters, then work is measured in newton-meters, and one newton-meter equals one *Joule*.

Definition. The *work* done by a variable force $F(x)$ directed along the x-axis from $x = a$ to $x = b$ is

$$W = \int_a^b F(x)\,dx.$$

Note. If we imagine partitioning the x-axis into a bunch of little dx slices, then the work done over one of these slices located at position x is then force $\times$ distance $= F(x)dx$. Integrating over all x values gives the definition above. (This is consistent with the usual informal interpretation of definite integrals as sums of dx [or dy] slices.)

Example. Page 452 number 8.

Recall. Hooke's Law states that the force it takes to stretch or compress a spring x length units from its natural (unstressed) length is proportional to x. That is, $F(x) = kx$ where k is the *spring constant* and carries unites of force per unit length.

Example. Page 452 number 4.

Note. We will consider several work problems which involve pumping a liquid out of a tank. We will describe the sides of the tanks by a function of y, take dy slices of the fluid, find the (1) volume of the slice, (2) weight of the slice, (3) work done on the slice, and integrate up the work done on the slices to find the total work. We must take dy slices since the only work done is force against the pull of gravity, and therefore motion along the y-axis.

Example. Page 455 number 35.