Chapter 7. Techniques of Integration

8.2 Integration by Parts

Theorem. (**Integration by Parts**)If $u = u(x)$ and $v = v(x)$ are differentiable functions of x, then we have

$$\int u\,dv = uv - \int v\,du.$$

Proof. By the Product Rule we have

$$\frac{d}{dx}[uv] = \left[\frac{du}{dx}\right] v + u\left[\frac{dv}{dx}\right].$$

Integrating both sides with respect to x and rearranging leads to the integral equation:

$$\begin{aligned}\int \left(u\frac{dv}{dx}\right) dx &= \int \left(\frac{d}{dx}[uv]\right) dx - \int \left(v\frac{du}{dx}\right) dx \\ &= uv - \int \left(v\frac{du}{dx}\right) dx.\end{aligned}$$

Q.E.D.

Note. Applying Integration by Parts to a definite integral, we have:

$$\int_{v_1}^{v_2} u\,dv = (u_2 v_2 - u_1 v_1) - \int_{u_1}^{u_2} v\,du.$$

In terms of areas, this gives the following figure.

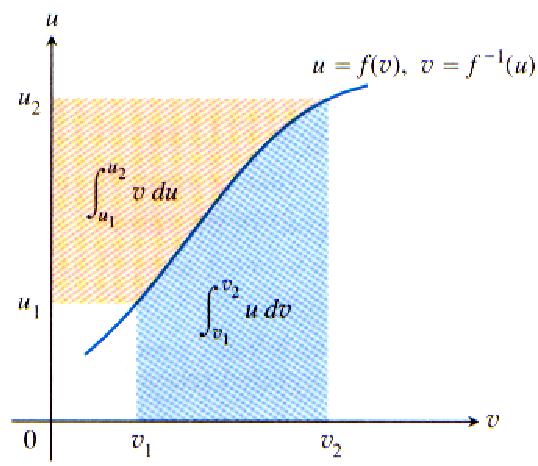

Figure 7.1 from Edition 10

Example. Page 563 Example 3. Evaluate $\int \ln x\,dx$.

Example. Page 564 Example 5. Evaluate

$$\int e^x \cos x\,dx.$$

(This is sort of weird!)

Example. Page 567 Example 9. Express $\int \cos^n x\,dx$ in terms of an integral of a lower power of $\cos x$. This is called a "reduction formula."

Examples. Page 568 numbers 4, page 569 numbers 34, page 570 number 42.