Chapter 9. Further Applications of Integration

9.5 Applications of First-Order Differential Equations

Note. In mechanics, it is common to assume that the force of resistance of a moving object is proportional to the velocity of the object. With velocity as v, mass as m and time as t, we have from Newton's law of motion,

$$\text{Force} = \text{mass} \times \text{acceleration}$$

or

$$m\frac{dv}{dt} = -kv \text{ or } \frac{dv}{dt} = -\frac{k}{m}v$$

where $k > 0$. As with all the above problems, we integrate to find that

$$v = v_0 e^{-(k/m)t}.$$

Example. Page 680 Number 2.

Note. The exponential growth model for population growth assumes infinite resources and unlimited growth that results from the growth rate constant. It is more realistic to assume that the environment has a *carrying capacity* M that represents the maximum population size which the environment can sustain in the long run. This leads us to the *logistic growth model.*

Definition. The differential equation

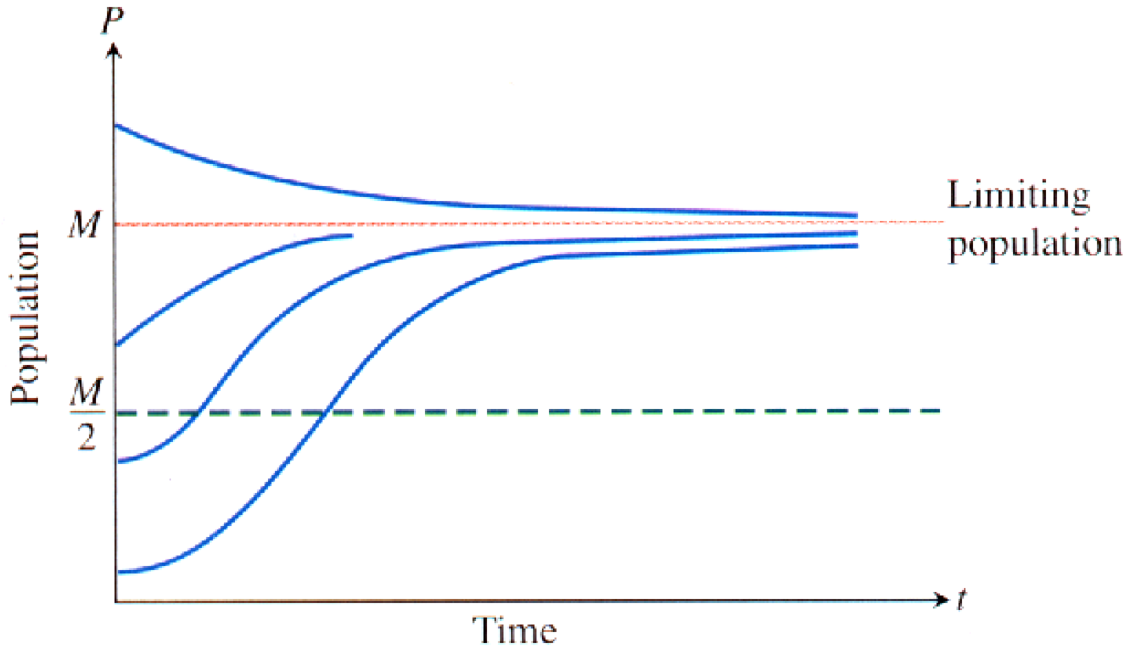
$$\frac{dP}{dt} = r(M - P)P$$

is the *logistic growth model.* The parameter r is the *logistic growth constant* and M is the carrying capacity.

Example. Page 681 number 10. Show that the general solution to the logistic equation is

$$P = \frac{M}{1 + Ae^{-rMt}}$$

where A is some constant (to be determined from initial population size). The graph of this function is



Page 676 Figure 9.24

Note. An *orthogonal trajectory* of a family of curves is a curve that intersects each curve of the family at right angles, or *orthogonally*.

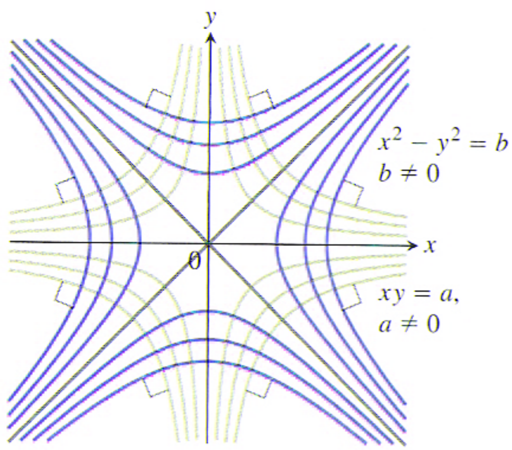


Figure 9.30, page 680.

Example. Page 681 Number 20b.