Calculus 2 Test 1 — Summer 2012

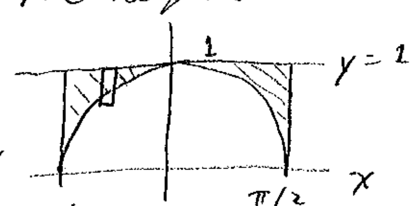
NAME KEY E-NUMBER ____________

SHOW ALL WORK!!! Do not rely on the calculators. Include all necessary symbols (such as equal signs and "+C" for indefinite integrals). When applicable, **draw the region** mentioned in the problem **and the resulting solid or surface.** The more details you show, the easier it will be to give you partial credit (if needed). Notice that some problems just ask you to *set up* the integrals for the solutions. Put your final answer in the box provided, or put a box around your final answer. Each numbered problem is worth 12 points.

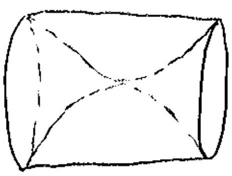
p372 #37

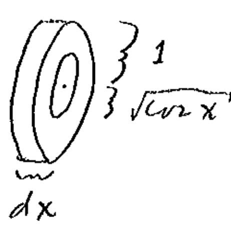
1. The region bounded above by $y = 1$ and below $y = \sqrt{\cos x}$ for $x \in [-\pi/2, \pi/2]$ is revolved about the x-axis. **Set up an integral** for the resulting volume.

The region is



Revolve about x-axis





For slice:

$$(\text{volume}) = \pi\left(\left(\begin{matrix}\text{outer}\\ \text{radius}\end{matrix}\right)^2 - \left(\begin{matrix}\text{inner}\\ \text{radius}\end{matrix}\right)^2\right)(\text{thickness})$$

$$= \pi\left((1)^2 - \left(\sqrt{\cos x}\right)^2\right)dx$$

$$= \pi\left(1 - \cos(x)\right)dx.$$

So, for solid:

$$\boxed{V = \int_{-\pi/2}^{\pi/2} \pi\left(1 - \cos x\right)dx}$$

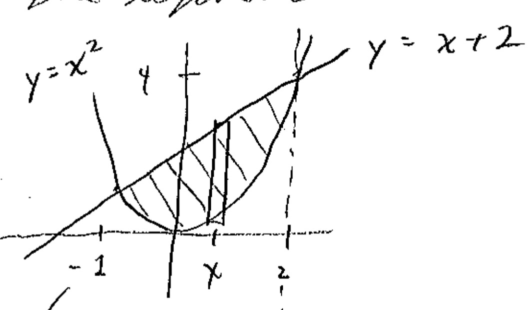
p380 #25a

2. The region bounded by $y = x + 2$ and $y = x^2$ is revolved about the line $x = 2$. **Set up an integral** for the resulting volume.

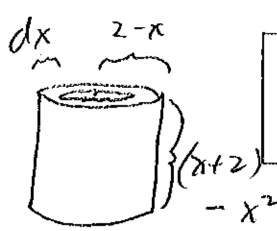
For slice: $(\text{volume}) = 2\pi(\text{radius})(\text{height})(\text{thickness})$

$$= 2\pi\,(2 - x)(x + 2 - x^2)\,dx$$

The region is



Revolve about x = 2



For solid

$$\boxed{V = \int_{-1}^{2} 2\pi(2 - x)(x + 2 - x^2)\,dx}$$

p386 #3

3. Set up an integral for the length of the curve $x = (y^3/3) + 1/(4y)$ for $y \in [1,3]$.

Well, $\frac{dx}{dy} = y^2 - \frac{1}{4y^2}$, so a differential of arclength is $ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \sqrt{1 + \left(y^2 - \frac{1}{4y^2}\right)^2}\, dy.$

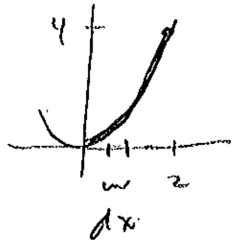
So for the total curve:

$$\boxed{L = \int_1^3 \sqrt{1 + \left(y^2 - \frac{1}{4y^2}\right)^2}\, dy}$$

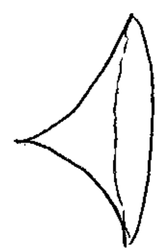
p391 #2

4. The curve $y = x^2$ for $x \in [0,2]$ is revolved about the x-axis. **Set up an integral** for the resulting surface area.

The curve is



Revolve about x-axis



$\}\ y = x^2$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + (2x)^2}\, dx$$

For a dx-slice:

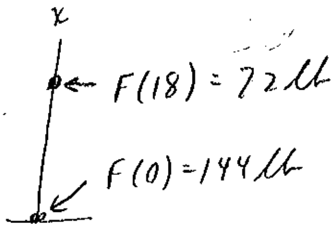
$$(\text{area}) = 2\pi(\text{radius})(\text{thickness})$$
$$= 2\pi\,(x^2)\, ds$$
$$= 2\pi\,(x^2)\sqrt{1 + (2x)^2}\, dx$$

So total length is

$$\boxed{L = \int_0^2 2\pi (x)^2 \sqrt{1 + (2x)^2}\, dx}$$

p 399 #8

5. A bag of sand originally weighing 144 lb was lifted at a constant rate. As it rose, sand also leaked out at a constant rate. The sand was half gone by the time the bag had been lifted to 18 ft. Express the weight F of the bag of sand as a function of height x. How much work was done lifting the sand to the height of 18 ft?



We know $F(0) = 144$ lb and $F(18) = 72$ lb AND the graph of F is a straight line ("sand leaked out at a constant rate"). So the slope of F is $\frac{144-72}{0-18} = \frac{-72}{18} = -4$. Hence $F(x) = -4x + 144$.

The work is $W = \int_0^{18} (-4x+144)\,dx = \left(-2x^2+144x\right)\Big|_0^{18}$ ft lb

$= \boxed{\left(-2(18)^2 + 144(18)\right) - 0} = 18(-36+144) = 18(108) = 1944$ ft lb

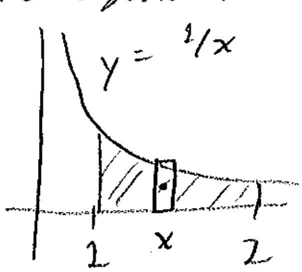
OK ANSWER!

$\boxed{1944 \text{ ft lb}}$

p 412 #9

6. Consider the region between the curve $y = 1/x$ and the x-axis for $x \in [1, 2]$. For this region, **set up integrals** for the moment about the x-axis (M_x), the moment about the y-axis (M_y), and the mass M. Express the center of mass of the region in terms of M_x, M_y, and M. Assume the region is of uniform density δ.

The region is



For a dx slice:

$(\text{area}) = \left(\frac{1}{x}\right)dx$

$(\text{mass}) = (\text{density})(\text{area})$

$= \delta \frac{1}{x}\,dx$

$\tilde{x} = x$

$\tilde{y} = \frac{1/x + 0}{2} = \frac{1}{2x}$

$\begin{pmatrix}\text{moment} \\ \text{about} \\ x\text{-axis}\end{pmatrix} = \tilde{y}\,(\text{mass}) = \frac{1}{2x}\,\delta\,\frac{1}{x}\,dx = \frac{\delta}{2x^2}\,dx$

$\begin{pmatrix}\text{moment} \\ \text{about} \\ y\text{-axis}\end{pmatrix} = \tilde{x}\,(\text{mass}) = x\,\delta\left(\frac{1}{x}\right)dx$

$= \delta\,dx$

So, for the region

$M = \int_1^2 \frac{\delta}{x}\,dx$

$M_x = \int_1^2 \frac{\delta}{2x^2}\,dx$

$M_y = \int_1^2 \delta\,dx$

$\bar{x} = \frac{M_y}{M}$

$\bar{y} = \frac{M_x}{M}$

p425 #6

7. (a) **Evaluate the indefinite integral** $\int \frac{\sec y \tan y \, dy}{2+\sec y}$.

let $u = 2 + \sec(y)$

$du = \sec(y) \tan(y) \, dy$

$= \int \frac{1}{u} \, du = \ln|u| + C = \ln|2+\sec(y)| + C$

$\boxed{\ln|2+\sec(y)|+C}$

p448 #3d

(b) Does the function $g(x) = (x+3)^2$ grow faster than, grow slower than, or grow at the same rate as $f(x) = x^2$? Justify your answer.

Consider $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{x^2}{(x+3)^2} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2x}{2(x+3)}$

$\overset{\infty/\infty}{=} \lim_{x\to\infty} \left(\frac{2}{2}\right) = 1$, so, f and g grow at the same rate.

$\boxed{\text{same rate}}$

p434 #18

8. Solve the separable differential equation: $\frac{dy}{dx} = \frac{e^{2x-y}}{e^{x+y}}$.

$\frac{dy}{dx} = \frac{e^{2x}e^{-y}}{e^{x}e^{y}} = \frac{e^{x}}{e^{2y}}$ or $e^{2y}dy = e^{x}dx$ and so

$\int e^{2y}dy = \int e^{x}dx \Rightarrow \frac{1}{2}e^{2y} = e^{x} + C$ or $\frac{1}{2}e^{2y} = e^{x} + k$.

Then $e^{2y} = 2(e^{x}+k)$

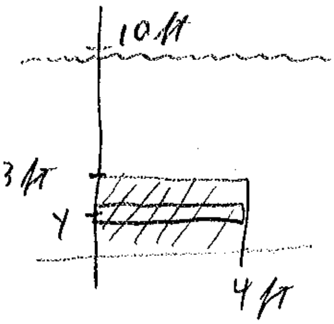
or $\ln(e^{2y}) = \ln(2(e^{x}+k))$

$2y = \ln(2(e^{x}+k))$ or $y = \frac{1}{2}\ln(2(e^{x}+k))$

$\boxed{y = \frac{1}{2}\ln(2(e^{x}+k))}$

p401 #35

Bonus. In a pool filled with water (of weight density 62.4 lb/ft^3) to a depth of 10 ft, calculate the fluid force on one side of a 3 ft by 4 ft rectangular plate if the plate rests vertically at the bottom of the pool on its 4-ft edge.



For a dy-slice: (area) = $4\,dy$ ft^2

(depth) = $10 - y$ ft

(pressure) = (weight density)(depth) = $62.4(10-y)$ lb/ft^2

(force) = (pressure)(area) = $62.4(10-y)\,4\,dy$ lb

So, for whole plate $F = \int_0^3 62.4(10-y)\,4\,dy = \int_0^3 249.6(10-y)\,dy$

$= 249.6 \int_0^3 (10-y)\,dy = 249.6\left(10y - \frac{1}{2}y^2\right)\Big|_0^3 = 249.6\left(10(3) - \frac{1}{2}(3)^2\right) - ($

$= 249.6\left(30 - \frac{9}{2}\right) = 249.6\left(\frac{51}{2}\right)$ lb

$$\boxed{249.6\left(\frac{51}{2}\right) \text{ lb}}$$