Page 16 Number 10.

Compute the linear combination $3\vec{u} + \vec{v} - \vec{w}$ where $\vec{u} = [1, 2, 1, 0]$, $\vec{v} = [-2, 0, 1, 6]$, and $\vec{w} = [3, -5, 1, -2]$.

Solution. We have

\[
3\vec{u} + \vec{v} - \vec{w} = 3[1, 2, 1, 0] + [-2, 0, 1, 6] - [3, -5, 1, -2]
\]

\[
= [3(1), 3(2), 3(1), 3(0)] + [-2, 0, 1, 6] - [3, -5, 1, -2]
\]

by Definition 1.1(3), "Scalar Multiplication"

\[
= [3 + (-2), 6 + 0, 3 + 1, 0 + 6] - [3, -5, 1, -2]
\]

by Definition 1.1(1), "Vector Addition"

\[
= [1, 6, 4, 6] - [3, -5, 1, -2] \text{ simplifying}
\]

\[
= [1 - (3), 6 - (5), 4 - (1), 6 - (-2)]
\]

by Definition 1.1(2), "Vector Subtraction"

\[
= [-2, 11, 3, 8] \text{ simplifying.}
\]

So we conclude $3\vec{u} + \vec{v} - \vec{w} = [-2, 11, 3, 8]$.

Page 16 Number 14.

Reproduce the vectors in this figure and draw an arrow representing $-3\vec{u} + 2\vec{w}$.

Solution. From Definition 1.1(3), "Scalar Multiplication," and the geometric interpretation of vectors (see the class notes, pages 2, 3, and 4) we represent $-3\vec{u}$ and $2\vec{w}$ as:

Then by the parallelogram property of addition:
Page 17 Number 40(a)

**Page 17 Number 40(a).** Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ and let $r, s$ be scalars in $\mathbb{R}$. Prove (A1): $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$.

**Proof.** Since $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$, by Definition 1.1, “Vectors in $\mathbb{R}^n$,” we have that $\vec{u} = [u_1, u_2, \ldots, u_n]$, $\vec{v} = [v_1, v_2, \ldots, v_n]$, and $\vec{w} = [w_1, w_2, \ldots, w_n]$ where all $u_i, v_i, w_i$ are real numbers. Then

$$(\vec{u} + \vec{v}) + \vec{w} = [(u_1, u_2, \ldots, u_n) + (v_1, v_2, \ldots, v_n)] + [w_1, w_2, \ldots, w_n]$$

$$= [u_1 + v_1, u_2 + v_2, \ldots, u_n + v_n] + [w_1, w_2, \ldots, w_n]$$

by Definition 1.1(1), “Vector Addition”

$$= [(u_1 + v_1) + w_1, (u_2 + v_2) + w_2, \ldots, (u_n + v_n) + w_n]$$

by Definition 1.1(1), “Vector Addition”

$$= [u_1 + (v_1 + w_1), u_2 + (v_2 + w_2), \ldots, u_n + (v_n + w_n)]$$

since addition of real numbers is associative.

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Page 17 Number 41(a)

**Page 17 Number 41(a).** Let $\vec{v}, \vec{w} \in \mathbb{R}^n$ and let $r$ be a scalar in $\mathbb{R}$. Prove (S1): $r(\vec{v} + \vec{w}) = r\vec{v} + r\vec{w}$.

**Proof.** Since $\vec{v}, \vec{w} \in \mathbb{R}^n$, by Definition 1.1, “Vectors in $\mathbb{R}^n$,” we have that $\vec{v} = [v_1, v_2, \ldots, v_n]$ and $\vec{w} = [w_1, w_2, \ldots, w_n]$ where all $v_i$ and $w_i$ are real numbers. Then

$$r(\vec{v} + \vec{w}) = r[(v_1, v_2, \ldots, v_n) + (w_1, w_2, \ldots, w_n)]$$

$$= [rv_1 + rw_1, rv_2 + rw_2, \ldots, rv_n + rw_n]$$

by Definition 1.1(1), “Vector Addition”

$$= r(v_1 + w_1), r(v_2 + w_2), \ldots, rv_n + w_n)]$$

by Definition 1.1(3), “Scalar Multiplication”

$$= [rv_1 + rv_2 + rv_3 + \ldots + rv_n + rw_1 + rw_2 + \ldots + rw_n]$$

since multiplication distributes over addition in the real numbers...
Page 16 Number 22. Find all scalars $c$ (if any) such that the vector $[c^2, -4]$ is parallel to the vector $[1, -2]$.

Solution. By Definition 1.2, two nonzero vectors are parallel if one is a scalar multiple of the other, say $[c^2, -4] = r[1, -2]$ for scalar $r \in \mathbb{R}$. Then by Definition 1.1(3), “Scalar Multiplication,” $[c^2, -4] = [r, -2r]$. So we need both $c^2 = r$ and $-4 = -2r$. Since $-4 = -2r$ then we must have $r = 2$. With $r = 2$ and $c^2 = r = 2$ we must have that either $c = \sqrt{2}$ or $c = -\sqrt{2}$. □

Page 16 Number 28. Find all scalars $c$ (if any) such that the vector $\vec{r} + c\vec{j} + (c - 1)\vec{k}$ is in the span of $\vec{r} + 2\vec{j} + \vec{k}$ and $3\vec{r} + 6\vec{j} + 3\vec{k}$.

Solution. By Definition 1.4, the span of $\vec{r} + 2\vec{j} + \vec{k}$ and $3\vec{r} + 6\vec{j} + 3\vec{k}$ is the set of all linear combinations of these two vectors. So the question becomes: For which $c \in \mathbb{R}$ is $\vec{r} + c\vec{j} + (c - 1)\vec{k} = r_1(\vec{r} + 2\vec{j} + \vec{k}) + r_2(3\vec{r} + 6\vec{j} + 3\vec{k})$ for some $r_1, r_2 \in \mathbb{R}$?

If this holds, $\vec{r} + c\vec{j} + (c - 1)\vec{k} = (r_1 + 3r_2)\vec{r} + (2r_1 + 6r_2)\vec{j} + (r_1 + 3r_2)\vec{k}$.

So we need $c \in \mathbb{R}$ such that

\begin{align*}
1 &= r_1 + 3r_2 \\
2 &= 2r_1 + 6r_2 \\
c - 1 &= r_1 + 3r_2
\end{align*}

Multiplying (1) by 2 gives $2 = 2r_1 + 6r_2$. Combining this with (2) we see that we need $c = 2$. With $c = 2$, equation (3) gives $1 = r_1 + 3r_2$ which is (1). Therefore all three equations (1), (2), and (3) are satisfied when $c = 2$. We can take $r_1 = 1$ and $r_2 = 0$, for example. □