SECTION 1-1
NUMBER 25

1.1.25 Find all scalars c such that the vector $[13, -15]$ is a linear combination of $[1,5]$ and $[3, c]$.

<u>Solution</u>

Well, a linear combination of these 2 vectors is a sum of the form:

$$r_1 [1, 5] + r_2 [3, c] \text{ for some } r_1, r_2 \in \mathbb{R}.$$

So we need scalar c such that

$$r_1 [1, 5] + r_2 [3, c] = [13, -15].$$

So, by definition of linear combination, we need $[r_1 + 3r_2, 5r_1 + r_2 c] = [13, -15]$; that is, we require:

$$r_1 + 3r_2 = 13 \qquad (1)$$

$$5r_1 + cr_2 = -15 \qquad (2)$$

From (1) we have $r_1 = 13 - 3r_2$ and then plugging this into (2) we have

$$5(13 - 3r_2) + cr_2 = -15$$

or $$65 - 15r_2 + cr_2 = -15$$

or $$cr_2 = -65 + 15r_2 - 15 = -80 + 15r_2$$

or $$c = \frac{-80 + 15r_2}{r_2} = \frac{-80}{r_2} + 15$$

<u>PLAN:</u> Let r_2 be ANY ("GOOD") real number. Then $c = (-80 + 15r_2)/r_2$ and $r_1 = 13 - 3r_2$.

Notice, we CAN'T have $r_2 \equiv 0$, so with $c = \frac{-80}{r_2} + 15$ then $c \neq 15$.

SECTION 1-1
NUMBER 25 (continued)

$\boxed{\text{So, } c \text{ can be any real number EXCEPT } 15.}$

Notice then that v_2 can correspondingly be anything but 0 and v_1 can correspondingly be anything but 13.