

SECTION 1.1
NUMBER 27 (like)

1.1.27 Find all $c \in \mathbb{R}$ such that $\hat{i} + c\hat{j} + 3\hat{k}$
is a linear combination of $\hat{i} + \hat{j}$ and $\hat{j} + 3\hat{k}$.

Solution

Well, the question is: For what $c \in \mathbb{R}$
we have $r_1, r_2 \in \mathbb{R}$ where

$$\hat{i} + c\hat{j} + 3\hat{k} = r_1(\hat{i} + \hat{j}) + r_2(\hat{j} + 3\hat{k})?$$

We need

$$\hat{i} + c\hat{j} + 3\hat{k} = r_1\hat{i} + (r_1 + r_2)\hat{j} + (3r_2)\hat{k},$$

or: \hat{i} condition: $1 = r_1$

\hat{j} condition: $c = r_1 + r_2$

\hat{k} condition: $3 = 3r_2$

So, we need $r_1 = 1$, $r_2 = 1$ and hence

$$\boxed{c = r_1 + r_2 = 1 + 1 = 2.} \quad \square$$

Note: In the book, we deal with

$$\hat{i} + c\hat{j} - 3\hat{k}. \quad \text{For this one } r_2 = -1 \text{ and } c = 0.$$

