

SECTION 1.1
EXERCISE #30

1.1.30 Find all $c \in \mathbb{R}$ such that $[c, -2c, c]$ is in the span of $[1, -1, 1]$, $[0, 1, -3]$, and $[0, 0, 1]$.

Solution

Well, the question is (see the definition of span): "Is $[c, -2c, c]$ a linear combination of the other three?"

That is, are there scalars r_1, r_2, r_3 such that:

$$\begin{aligned} [c, -2c, c] &= r_1 [1, -1, 1] + r_2 [0, 1, -3] + r_3 [0, 0, 1] \\ &= [r_1, -r_1 + r_2, r_1 - 3r_2 + r_3] ? \end{aligned}$$

So we consider

$$c = r_1 \quad (1)$$

$$-2c = -r_1 + r_2 \quad (2)$$

$$c = r_1 - 3r_2 + r_3 \quad (3)$$

From (1) and (3) we have: $r_1 = r_1 - 3r_2 + r_3$

$$\text{or } 0 = -3r_2 + r_3 \text{ or } r_3 = 3r_2.$$

From (2) and (3) we have (adding (1) to (2)):

$$-c = r_2. \text{ Notice then that } r_3 = 3r_2 = 3(-c) = -3c.$$

So for ANY given $c \in \mathbb{R}$, we can take

$r_1 = c$, $r_2 = -c$, and $r_3 = -3c$ and we will have $[c, -2c, c]$ in the span of the other three vectors. SO

c can be any scalar.