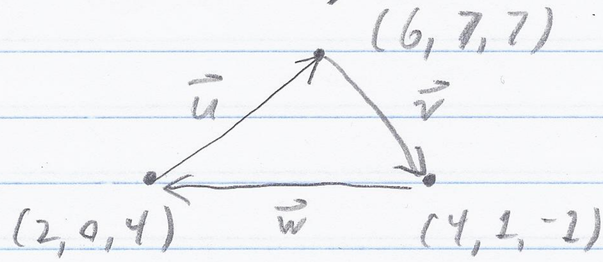
SECTION 1.2
NUMBER 23

1.2.23 Prove that the points $(2,0,4)$, $(4,1,-1)$, and $(6,7,7)$ are vertices of a right triangle in $\mathbb{R}^3$.

<u>Solution</u>

Schematically we have:



Then
$$\vec{u} = [6-2, 7-0, 7-4] = [4, 7, 3]$$
$$\vec{v} = [4-6, 1-7, -1-7] = [-2, -6, -8]$$
$$\vec{w} = [2-4, 0-1, 4-(-1)] = [-2, -1, 5].$$

We need to consider DOT PRODUCTS to see if two of these vectors are orthogonal (see class notes §1.2 page 4). Well:

$$\vec{u} \cdot \vec{v} = [4, 7, 3] \cdot [-2, -6, -8]$$
$$= (4)(-2) + (7)(-6) + (3)(-8) = -8 - 42 - 24$$
$$= -74 \neq 0$$

$$\vec{v} \cdot \vec{w} = [-2, -6, -8] \cdot [-2, -1, 5]$$
$$= (-2)(-2) + (-6)(-1) + (-8)(5) = 4 + 6 - 40$$
$$= -30 \neq 0$$

$$\vec{u} \cdot \vec{w} = [4, 7, 3] \cdot [-2, -1, 5]$$
$$= (4)(-2) + (7)(-1) + (3)(5) = -8 - 7 + 15 = 0$$

So $\vec{u} \perp \vec{w}$ and $\boxed{\text{YES}}$ these points form a right triangle. $\square$