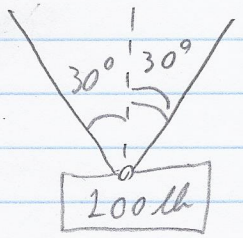
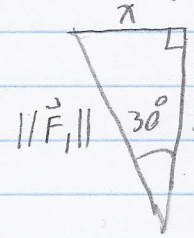
SECTION 2.2
EXERCISE #37

2.2.37 A 100 lb weight is suspended by a rope passed through an eyelet on top of the weight and making angles of 30° with the vertical: Find the tension (magnitude of the force vector) along the rope.



Solution

Let the force vectors be $\vec{F}_1$ and $\vec{F}_2$ along the rope. The upward component of $\vec{F}_1 + \vec{F}_2$ must be 100 lb to balance the downward weight of 100 lb. So, by symmetry, the upward component of both $\vec{F}_1$ and $\vec{F}_2$ must be 50 lb. So we can find the components of $\vec{F}_1$ (and of $\vec{F}_2$) from a right triangle:



$$\text{So } \tan(30°) = \frac{x}{50\text{ lb}} \text{ or } \frac{1}{\sqrt{3}} = \frac{x}{50\text{ lb}} \text{ or } x = \frac{50}{\sqrt{3}}\text{ lb}.$$

Hence $\vec{F}_1 = \left[\frac{-50}{\sqrt{3}}\text{ lb}, 50\text{ lb}\right]$ and $\vec{F}_2 = \left[\frac{50}{\sqrt{3}}\text{ lb}, 50\text{ lb}\right]$.

The tension in the rope is the magnitude of a force vector:

$$\|\vec{F}_1\| = \|\vec{F}_2\| = \sqrt{\left(\frac{\pm 50}{\sqrt{3}}\text{ lb}\right)^2 + (50\text{ lb})^2} = 50\sqrt{\frac{1}{3}+1}\text{ lb}$$

$$= 50\sqrt{\frac{4}{3}}\text{ lb} = \boxed{\frac{100}{\sqrt{3}}\text{ lb}.} \qquad \square$$