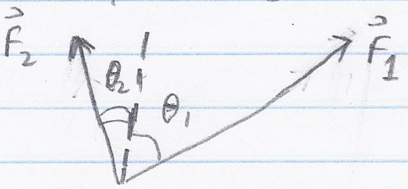
SECTION 1.2

EXERCISE #39

1.2.39 Suppose that a weight of 100 lb is suspended by two different ropes tied at an eyelet on top of the weight. Let the angles the ropes make with the vertical be θ_1 and θ_2:

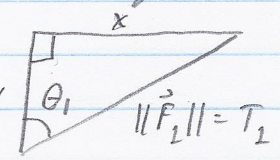


Let the tensions in the ropes be T_1 for the right-hand rope and T_2 for the left-hand rope.

(a) Show that the force vector $\vec{F}_1$ is
$$\vec{F}_1 = (T_1 \sin\theta_1)\hat{\imath} + (T_1 \cos\theta_1)\hat{\jmath}.$$

Solution

We use a right triangle:



So $\cos\theta_1 = \dfrac{y}{T_1}$ and $\sin\theta_1 = \dfrac{x}{T_1}$.

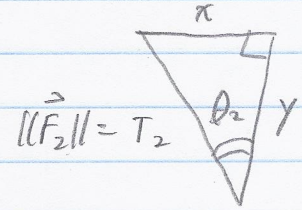
Then
$$y = T_1\cos\theta_1 \text{ and } x = T_1 \sin\theta_1.$$

So $\vec{F}_1 = x\hat{\imath} + y\hat{\jmath} = (T_1 \sin\theta_1)\hat{\imath} + (T_1\cos\theta_1)\hat{\jmath}.$

(b) Find the corresponding expression for $\vec{F}_2$ in terms of T_2 and θ_2.

Solution

As in part (a),



so $\cos\theta_2 = \dfrac{y}{T_2}$ and $\sin\theta_2 = \dfrac{x}{T_2}$.

Then
$$y = T_2\cos\theta_2 \text{ and } x = T_2 \sin\theta_2.$$
So (watch the signs!) $\boxed{\vec{F}_2 = -x\hat{\imath} + y\hat{\jmath} = (-T_2\sin\theta_2)\hat{\imath} + (T_2\cos\theta_2)\hat{\jmath}.}$

SECTION 1.2
EXERCISE #39 (continued)

(c) If the system is in equilibrium, $\vec{F}_1 + \vec{F}_2 = 100\hat{j}$ lb, so $\vec{F}_1 + \vec{F}_2$ must have $\hat{i}$-component 0 lb and $\hat{j}$-component 100 lb. Write two equations reflecting this fact, using the answers to parts (a) and (b).

Solution

From parts (a) and (b) we have

$$\vec{F}_1 + \vec{F}_2 = (T_1 \sin\theta_1 - T_2 \sin\theta_2)\hat{i} + (T_1 \cos\theta_1 + T_2 \cos\theta_2)\hat{j}.$$

So we need

$$\boxed{\begin{aligned} T_1 \sin\theta_1 - T_2 \sin\theta_2 &= 0 \text{ lb, and} \\ T_1 \cos\theta_1 + T_2 \cos\theta_2 &= 100 \text{ lb.} \end{aligned}}$$

□

(d) Find T_1 and T_2 if $\theta_1 = 45°$ and $\theta_2 = 30°$.

Solution

Since $\cos(45°) = \sin(45°) = 1/\sqrt{2}$, $\cos(30°) = \sqrt{3}/2$, and $\sin(30°) = 1/2$, then part (c) gives the equations

$$T_1\left(\frac{1}{\sqrt{2}}\right) - T_2\left(\frac{1}{2}\right) = 0 \text{ lb} \quad (1)$$

$$T_1\left(\frac{1}{\sqrt{2}}\right) + T_2\left(\frac{\sqrt{3}}{2}\right) = 100 \text{ lb} \quad (2).$$

Subtracting (1) from (2) we have $\frac{\sqrt{3}+1}{2} T_2 = 100$ lb

or $T_2 = \frac{200}{\sqrt{3}+1}$ lb. Substituting this into (1) gives

$$T_1\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{200}{\sqrt{3}+1}\right)\left(\frac{1}{2}\right) \text{ lb} = 0 \text{ lb} \quad \text{or} \quad \frac{1}{\sqrt{2}} T_1 = \frac{100}{\sqrt{3}+1} \text{ lb}$$

or $T_1 = \frac{100\sqrt{2}}{\sqrt{3}+1}$ lb. So the solution is

$$\boxed{T_1 = \frac{100\sqrt{2}}{\sqrt{3}+1} \text{ lb and } T_2 = \frac{200}{\sqrt{3}+1} \text{ lb.}}$$

□