

SECTION 1.3
NUMBER 37

1.3.37 The Hilbert matrix H_n is $n \times n$ matrix $[h_{ij}]$ where $h_{ij} = \frac{1}{i+j-1}$. Prove

this matrix is symmetric.

Proof

Recall that square matrix $A = [a_{ij}]$ is symmetric if $A = A^T = [a_{ij}]^T = [a_{ji}]$.

Here we have

$$\begin{aligned} [h_{ij}]^T &= [h_{ji}] = \left[\frac{1}{(j)+(i)-1} \right] \\ &= \left[\frac{1}{i+j-1} \right] = [h_{ij}]. \end{aligned}$$

Therefore $[h_{ij}]$ is symmetric. ■