

SECTION 1.3
EXERCISE #45

1, 3, 45 Find all values of r for which

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix} \text{ commutes with } \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Solution

We compute the two relevant products:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (2)(1) + (0)(0) + (0)(1) & (2)(0) + (0)(1) + (0)(0) & (2)(1) + (0)(0) + (0)(1) \\ (0)(1) + (1)(0) + (0)(1) & (0)(0) + (1)(1) + (0)(0) & (0)(1) + (1)(0) + (0)(1) \\ (0)(1) + (0)(0) + (r)(1) & (0)(0) + (0)(1) + (r)(0) & (0)(1) + (0)(0) + (r)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ r & 0 & r \end{bmatrix}, \text{ and}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix}$$

$$= \begin{bmatrix} (1)(2) + (0)(0) + (1)(0) & (1)(0) + (0)(1) + (1)(0) & (1)(0) + (0)(0) + (1)(r) \\ (0)(2) + (1)(0) + (0)(0) & (0)(0) + (1)(1) + (0)(0) & (0)(0) + (1)(0) + (0)(r) \\ (1)(2) + (0)(0) + (1)(0) & (1)(0) + (0)(1) + (1)(0) & (1)(0) + (0)(0) + (1)(r) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & r \\ 0 & 1 & 0 \\ 2 & 0 & r \end{bmatrix}.$$

Comparing corresponding entries of the products we need $2 = r$ (for (1,3) entry), $r = 2$ (for (3,1) entry), and $r = r$ (for (3,3) entry). So the matrices commute for $r = 2$. \square

Notice the error in the answer in the back of the book!