SECTION 1.4

NUMBER 15 (modified)

1.4.15 Find all solutions using the Gauss-Jordan method:

$$\begin{aligned} y + z &= 6 \\ 3x - y + z &= -7 \\ x + y - 3z &= -13. \end{aligned}$$

Solution

Well, lets create the augmented matrix $[A|\vec{b}]$ and reduce it to reduced row echelon form:

$$\begin{bmatrix} 0 & 1 & 1 & 6 \\ 3 & -1 & 1 & -7 \\ 1 & 1 & -3 & -13 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} \boxed{1} & 1 & -3 & -13 \\ \textcircled{3} & -1 & 1 & -7 \\ 0 & 1 & 1 & 6 \end{bmatrix}$$

$$\xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 1 & 1 & -3 & -13 \\ 0 & -4 & 10 & 32 \\ 0 & 1 & 1 & 6 \end{bmatrix} \xrightarrow{R_2 \to R_3} \begin{bmatrix} \boxed{1} & \textcircled{1} & -3 & -13 \\ 0 & \boxed{1} & 1 & 6 \\ 0 & \textcircled{-4} & 10 & 32 \end{bmatrix}$$

$$\begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + 4R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -4 & -19 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 14 & 56 \end{bmatrix} \xrightarrow{R_3 \to R_3/(14)} \begin{bmatrix} \boxed{1} & 0 & \textcircled{-4} & -19 \\ 0 & \boxed{1} & \textcircled{1} & 6 \\ 0 & 0 & \boxed{1} & 4 \end{bmatrix}$$

$$\begin{matrix} R_1 \to R_1 + 4R_3 \\ R_2 \to R_2 - R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}.$$

$\boxed{\text{Ie } x = -3, \; y = 2, \text{ and } z = 4.}$ Let's check it:

$$y + z = (2) + (4) = 6 \checkmark$$

$$3x - y + z = 3(-3) - (2) + (4) = -7 \checkmark$$

$$x + y - 3z = (-3) + (2) - 3(4) = -13 \checkmark. \quad \square$$