

SECTION 1.4

NUMBER 17

1.4.17 Solve (i.e., find the general solution)

$$\text{for } x_1 - 2x_2 = 3$$

$$3x_1 - x_2 = 14$$

$$x_1 - 7x_2 = -2.$$

Solution

The augmented matrix is:

$$[A|\vec{b}] = \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 3 & -1 & 14 \\ 1 & -7 & -2 \end{array} \right] \begin{array}{l} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 5 & 5 \\ 0 & -5 & -5 \end{array} \right]$$

$$\underline{R_3 \rightarrow R_3 + R_2} \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 5 & 5 \\ 0 & 0 & 0 \end{array} \right] \underline{R_2 \rightarrow R_2 / 5} \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$$

$$\underline{R_1 \rightarrow R_1 + 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]. \quad \text{So, } \boxed{\begin{array}{l} x_1 = 5 \\ x_2 = 1 \end{array}} \quad \square$$