SECTION 1.4
EXERCISE #24

1.4.24 Use Gauss-Jordan method to solve:

$$x_1 + 2x_2 - 3x_3 + x_4 = 2$$
$$3x_1 + 6x_2 - 8x_3 - 2x_4 = 1.$$

Solution

Well, the augmented matrix is

$$\left[\begin{array}{cccc|c} \boxed{1} & 2 & -3 & 1 & 2 \\ \textcircled{3} & 6 & -8 & -2 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 - 3R_1} \left[\begin{array}{cccc|c} 1 & 2 & \textcircled{-3} & 1 & 2 \\ 0 & 0 & \boxed{1} & -5 & -5 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + 3R_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & -14 & -13 \\ 0 & 0 & 1 & -5 & -5 \end{array}\right] = [H|\vec{c}]$$

The corresponding system of equations for augmented matrix $[H|\vec{c}]$ is

$$x_1 + 2x_2 - 14x_4 = -13$$
$$x_3 - 5x_4 = -5$$

or

$$x_1 = -13 - 2x_2 + 14x_4$$
$$x_2 = x_2$$
$$x_3 = -5 + 5x_4$$
$$x_4 = x_4$$

So, let the free variables be $r = x_2$ and $s = x_4$ to get:

$$\boxed{\begin{aligned} x_1 &= -13 - 2r + 14s \\ x_2 &= r \\ x_3 &= -5 + 5s \\ x_4 &= s \end{aligned}}$$

OR

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -13 \\ 0 \\ -5 \\ 0 \end{bmatrix} + r\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 14 \\ 0 \\ 5 \\ 1 \end{bmatrix}. \qquad \square$$