

SECTION 1.4  
NUMBER 9

2.4.9 Describe all solutions of a linear system with augmented matrix:

$$\left[ \begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

Solution

The corresponding system of equations is

$$x_1 + 2x_3 = 1 \quad \text{This gives}$$

$$x_2 + x_3 + 3x_4 = -2$$

$$0 = 0.$$

$$x_1 = 1 - 2x_3$$

$$x_2 = -2 - x_3 - 3x_4$$

$$0 = 0.$$

Let  $r = x_3$  and

$s = x_4$  be FREE

VARIABLES!

Then

$$x_1 = 1 - 2r$$

$$x_2 = -2 - r - 3s$$

$$x_3 = r$$

$$x_4 = s$$

for any  $r, s \in \mathbb{R}$

OR

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

for any  $r, s \in \mathbb{R}$ .

Notice that if (or requested)  $x_3 = 3$  and

$$x_4 = -2 \quad \text{then} \quad x_1 = -5 \quad \text{and} \quad x_2 = 1.$$

$$\leftarrow 1 - 2(3) \quad (2 - (3) - 3(-2))$$