SECTION 1.6
EXERCISE #25

1.6.25 Is $\{[2,1,-3],[4,0,2],[2,-1,3]\}$ a basis for a subspace of $\mathbb{R}^3$ that it spans?

<u>Solution</u>

Let's use Theorem 1.16 and put these 3 vectors in as <u>columns</u> of matrix A:

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 0 & -1 \\ -3 & 2 & 3 \end{bmatrix}.$$ Notice

$$A \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 4 & 2 \\ -3 & 2 & 3 \end{bmatrix} \xrightarrow[R_3 \to R_3 + 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 4 & 4 \\ 0 & 2 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 4 & 4 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow[R_3 \to R_3/4]{R_2 \to R_2/2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

So $A \sim I$ and by Theorem 1.16 (see parts 2 and 4), the columns of A form a basis for $\mathbb{R}^3$. That is, $\boxed{\text{YES}}$ the 3 given vectors are a basis for the subspace of $\mathbb{R}^3$ which they span (in fact, they span $\mathbb{R}^3$). $\square$