SECTION 2.6
EXERCISE #5

2.6.5 Is $\{[x, y, z] \mid x, y, z \in \mathbb{R} \text{ and } z = 3x + 2\}$ a subspace of $\mathbb{R}^3$?

Solution

Well, the set is a subspace of $\mathbb{R}^3$ if it is closed under vector addition and scalar multiplication (see Definition 1.16).

Let's test for closure under scalar multiplication first. Let $\vec{v} = [x_1, y_1, z_1]$ and $r \in \mathbb{R}$. Then $x_1, y_1, z_1 \in \mathbb{R}$ and $z_1 = 3x_1 + 2$. Consider

$$r\vec{v} = r[x_1, y_1, z_1] = [rx_1, ry_1, rz_1]$$

BUT does $(rz_1) = 3(rx_1) + 2$?

Well, $z_1 = 3x_1 + 2$, so we need

$$rz_1 = r(3x_1 + 2) = 3(rx_1) + 2$$

$$\text{or } 3rx_1 + 2r = 3rx_1 + 2$$

$$\text{or } 2r = 2.$$

The last equation implies that $r = 1$, but this must hold for ALL $r \in \mathbb{R}$, so the is NOT closed under scalar multiplication!

SO, **NO** the set is NOT a subspace. □