

2.1.13 Find a basis for the row space of the matrix:

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 4 & 2 \\ 3 & 2 & 8 & 7 \end{bmatrix}.$$

Solution

The row space of A is the span of the rows of A :

$$\text{Row space} = \text{sp}([1, 3, 5, 7], [2, 0, 4, 2], [3, 2, 8, 7]).$$

According to Theorem 2.1.4, to find a basis for the span of a set of vectors, we put those vectors in a matrix as COLUMNS, and then row reduce. So we have

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 2 \\ 5 & 4 & 8 \\ 7 & 2 & 7 \end{bmatrix} \xrightarrow{\text{row}} \begin{bmatrix} \boxed{1} & 0 & 2/3 \\ 0 & \boxed{1} & 7/6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = H.$$

Since H has pivots in the first and second columns, then the first and second columns of matrix B form a basis for the space which they span. So (as rows of A) a basis for the row space of A is

$$\boxed{\{[1, 3, 5, 7], [2, 0, 4, 2]\}}. \quad \square$$