SECTION 2.1
EXERCISE 29

2.1.29 Let $\vec{u}$ and $\vec{v}$ be two different vectors in $\mathbb{R}^n$.
Prove that $\{\vec{u}, \vec{v}\}$ is linearly dependent if and only if one of the vectors is a multiple of the other.

Proof

Suppose $\{\vec{u}, \vec{v}\}$ is linearly dependent. Then $r_1\vec{u} + r_2\vec{v} = \vec{0}$ where either r_1 or r_2 is nonzero. If $r_1 \neq 0$ then $\vec{u} = (-r_2/r_1)\vec{v}$; if $r_2 \neq 0$ then $\vec{v} = (-r_1/r_2)\vec{u}$. In either case, one of the vectors is a multiple of the other.

Suppose one of the vectors is a multiple of the other, say $\vec{u} = r\vec{v}$. Then $1\vec{u} + (-r)\vec{v} = \vec{0}$. Since the coefficient of $\vec{u}$ is not 0 then this is a dependence relation and $\{\vec{u}, \vec{v}\}$ is linearly dependent. ■