SECTION 2.1

NUMBER 33

2.1:33 Find all scalars s, if any exist, such that $[1, 0, 1]$, $[2, s, 3]$, $[1, -s, 0]$ are independent.

<u>Solution</u>

We consider the equation

$r_1 [1, 0 1] + r_2 [2, s, 3] + r_3 [1, -s, 0] = [0, 0, 0]$

and test to see if s exists where some $r_i \neq 0$ for $i = 1, 2, 3$. (see Definition 2.1).

We get

$[r_1 + 2r_2 + r_3, sr_2 - sr_3, r_1 + 3r_2] = [0, 0, 0]$,

and so we must have

$$\begin{aligned} r_1 + 2r_2 + r_3 &= 0 \\ sr_2 - sr_3 &= 0 . \\ r_1 + 3r_2 &= 0 \end{aligned}$$

Notice that if $s = 0$ then the second equation becomes $0 = 0$ and we can take $r_1 = 3$, $r_2 = -1$, $r_3 = -1$ (for example; there are an infinite number of solutions in fact).

If $s \neq 0$, we can divide both sides of the second equation by s to get $r_2 - r_3 = 0$. The associated augmented matrix is then

$$[A \mid \vec{0}] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 3 & 0 & 0 \end{array}\right] \underset{\text{RREF}}{\overset{\text{We}}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = [H \mid \vec{0}]$$

Since H is not row equivalent to the identity, then there are infinitely many solutions to $A\vec{x} = \vec{0}$ (there is a free variable; see Corollary 2.(2) of § 1.6).

So there is a nontrivial solution where some

SECTION 2.1

NUMBER 33 (continued)

$r_i \neq 0$ for $i = 1, 2, 3$. So whether $s = 0$ or $s \neq 0$, there are values for r_1, r_2, r_3 (not all 0) for which

$r_1[1, 0, 1] + r_2[2, s, 3] + r_3[1, -s, 0] = [0, 0, 0]$.

That is, the vectors are dependent no matter the choice of s. So there is $\boxed{\text{no scalar } s \text{ making the vectors independent.}}$

□