SECTION 2.1
EXERCISE # 5

2.1.5 Give a geometric criterion for a set of three distinct nonzero vectors in $\mathbb{R}^3$ to be dependent.

<u>Solution</u>

Well, if the three vectors are <u>independent</u> then (by Theorem 2.3(2)) the vectors form a basis for $\mathbb{R}^3$. So if they are dependent then they don't form a basis of $\mathbb{R}^3$ but they span a SUBSPACE of $\mathbb{R}^3$ (of dimension 1 or 2). The Note on page 2 of the class notes for this section implies that a 1-dimensional subspace of $\mathbb{R}^n$ is a line and a 2-dimensional subspace of $\mathbb{R}^n$ is a plane. BUT for this interpretation, we want the vectors to all be in <u>standard position</u>!

So, geometrically, the 3 distinct nonzero vectors in $\mathbb{R}^3$ must lie (when in <u>standard position</u>) either along a line which passes through the origin OR lie in a plane that passes through the origin.