

SECTION 2.2
EXERCISE #19

2.2.19

In Exercise #18 it proved that $\text{rank}(AC) \leq \text{rank}(A)$.
Give an example where $\text{rank}(AC) < \text{rank}(A)$.

Solution

We need to choose a matrix C that "destroys" columns of matrix A . For an example of size

2×2 let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Then

$\text{rank}(A) = 2$ and $AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ so that

$\text{rank}(AC) = 1$. This example suffices, but for an $n \times n$ example we can let A be the $n \times n$ identity matrix so that $\text{rank}(A) = n$.

Then define C such that the first k columns of C are the same as the first k columns of A (where $1 \leq k \leq n-1$) and the remaining $n-k$ columns of C are the zero vector $\vec{0} \in \mathbb{R}^n$.

Then $\text{rank}(C) = k$ and $\text{rank}(AC) = \text{rank}(C) = k$ so that $k = \text{rank}(AC) < \text{rank}(A) = n$. \square