

SECTION 2.2  
EXERCISE # 21

2.2.21 Let  $A$  be an  $m \times n$  matrix. Prove that  $\text{rank}(AA^T) = \text{rank}(A)$ .

Proof

The proof is easy with the hint given in the text (and probably impossible at this stage without it). We are given that  $\text{rank}(A^T A) = \text{rank}(A)$ .

The column space of  $B$  is the row space of  $B^T$  so  $\text{rank}(B) = \text{rank}(B^T)$ . With  $B = A^T$  and  $B^T = (A^T)^T = A$  we have

$$\text{rank}(AA^T) = \text{rank}(B^T B)$$

$$= \text{rank}(B) \text{ by the hint (which holds for all matrices and so holds for matrix } B)$$

$$= \text{rank}(B^T) = \text{rank}(A), \quad \blacksquare$$

$$= \text{rank}(A)$$