SECTION 2.2
NUMBER 3

2.2.3 Consider $A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}$.

Find (a) rank, (b) basis for row space, (c) basis for column space, and (d) basis for nullspace.

<u>Solution</u>

Notice $A \underset{RREF}{\overset{We}{\sim}} \begin{bmatrix} \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 \end{bmatrix} = H$

SO:

(a) $\boxed{\text{rank}(A) = 3}$ since H has 3 pivots.

(b) A basis for the row space is given by the <u>nonzero rows of H</u> (by Note 2.2.A(2)): $\boxed{\{[1,0,-1,0], [0,1,1,0], [0,0,0,1]\}}$.

(c) A basis for the column space of C is given by the <u>columns of C</u> which correspond to pivot containing columns of H:

$$\boxed{\left\{ \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \end{bmatrix} \right\}}.$$

(d) For the nullspace we consider solutions to $A\vec{x} = 0$ or, equivalently, $H\vec{x} = \vec{0}$. With $\vec{x} = [x_1, x_2, x_3, x_4]^T$ we have from $H\vec{x} = \vec{0}$:

SECTION 2.2

NUMBER 3 (continued)

$$\begin{aligned} x_1 \quad - x_3 &= 0 \\ x_2 + x_3 &= 0 \\ x_4 &= 0 \\ 0 &= 0 \end{aligned} \quad \text{or} \quad \begin{aligned} x_1 &= x_3 \\ x_2 &= -x_3 \\ x_3 &= x_3 \\ x_4 &= 0 \end{aligned}.$$

With $r = x_3$ as a free variable we have

$$\begin{aligned} x_1 &= r \\ x_2 &= -r \\ x_3 &= r \\ x_4 &= 0 \end{aligned} \quad \text{or} \quad \vec{x} = r\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \text{ where } r \in \mathbb{R}.$$

So a basis for the nullspace is $\boxed{\left\{\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}\right\}}$. $\square$