SECTION 2.2
EXERCISE #7

2.2.7 Is this matrix invertible? Base your answer on the rank of the matrix

$$A = \begin{bmatrix} 0 & -9 & -9 & 2 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

Solution

Let's use Theorem 2.6 ("An invertibility Criterion"): If $\text{rank}(A) = 4$ then A is invertible (and if $\text{rank}(A) < 4$ then A is not invertible). Well

$$A \xrightarrow{\text{RREF}} \text{Wa} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = H.$$

(Wolfram Alpha)

PIVOTS

Since H has 3 pivots then $\text{rank}(A) = 3$.

So, $\boxed{A \text{ is NOT invertible.}}$ □