

SECTION 2.3
EXERCISE #23

2.3.23 Find a formula for $T^{-1}(\vec{x})$ where T is as given in Exercise #15.

Solution.

Well, the standard matrix representation of T^{-1} is A^{-1} where we found A in #15:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Wolfram Alpha says $A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}.$

Now $T^{-1}\vec{x} = A^{-1}\vec{x}$ so consider

$$A^{-1}\vec{x} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_2 - x_3 \\ x_1 - x_2 \end{bmatrix}.$$

So a formula for T^{-1} is (as a row vector):

$$T^{-1}([x_1, x_2, x_3]) = [x_3, x_2 - x_3, x_1 - x_2].$$