

SECTION 2.3
EXERCISE #5

2.3.5 If $T([1, 0]) = [3, -1]$ and $T([0, 1]) = [-2, 5]$.
Find $T([4, -6])$. Here, T is linear!

Solution

Well, notice that $[4, -6] = 4[1, 0] - 6[0, 1]$,
so

$$\begin{aligned} T([4, -6]) &= T(4[1, 0] - 6[0, 1]) \\ &= T(4[1, 0]) - T(6[0, 1]) \quad \text{since } T \text{ is linear} \\ &= 4T([1, 0]) - 6T([0, 1]) \quad \text{since } T \text{ is linear} \\ &= 4[3, -1] - 6[-2, 5] \\ &= [12, -4] - [-12, 30] = \boxed{[24, -34]}. \quad \square \end{aligned}$$