SECTION 2.5
NUMBER 19

2.5.19 (like) Find parametric equations for the line in $\mathbb{R}^5$ containing points $(2,1,3,4,0)$ and $(1,2,-2,3,-1)$.

<u>Solution</u>

By Definition 2.4/2.5, for the desired line L, we need a direction vector $\vec{d}$ along line L. Let's use the vector from one point to the other, so

$$\vec{d} = [2-1,\ 1-2,\ 3-(-1),\ 4-3,\ 0-(-1)]$$
$$= [1, -1, 4, 1, 1].$$

For the translation vector $\vec{a}$, let's use a vector from the "origin" to one of the points, say

$$\vec{a} = [2-0,\ 1-0,\ 3-0,\ 4-0,\ 0-0]$$
$$= [2, 1, 3, 4, 0].$$

So the desired line is $\vec{x} = t\vec{d} + \vec{a}$, so

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = t\begin{bmatrix} 1 \\ -1 \\ 4 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 3 \\ 4 \\ 0 \end{bmatrix} \quad \text{where } t \in \mathbb{R}, \text{ so}$$

$$\boxed{\begin{array}{ll} \vec{x}_1 = t + 2 & x_4 = t + 4 \\ \vec{x}_2 = -t + 1 & x_5 = t \\ x_3 = 4t + 3 & \end{array}}.$$

(continued →)

SECTION 2.5
NUMBER 19 (continued)

Notice that the question in the book asks for the midpoint of the section joining the two points. We can use the answer above to solve this by observing that when $t = {}^-1$ we have $x_1 = (^-1) + 2 = 1$ and when $t = 0$, $x_1 = 2$

$x_2 = -(^-1) + 1 = 2$ $\quad x_2 = 1$

$x_3 = 4(^-1) + 3 = ^-1$ $\quad x_3 = 3$

$x_4 = (^-1) + 4 = 3$ $\quad x_4 = 4$

$x_5 = (^-1) + 0 = ^-1$ $\quad x_5 = 0$.

That is, when $t = -1$ we get the point $(1, 2, ^-1, 3, ^-1)$ and when $t = 0$ we get the point $(2, 1, 3, 4, 0)$. So the midpoint of the line segment joining these can be found at $t = (^-1 + 0)/2 = {}^-1/2$ and is

$x_1 = (^-1/2) + 2 = 3/2$

$x_2 = -(^-1/2) + 1 = 3/2$

$x_3 = 4(^-1/2) + 3 = 1$

$x_4 = (^-1/2) + 4 = 7/2$

$x_5 = (^-1/2) + 0 = ^-1/2$

or $\boxed{(3/2, 3/2, 1, 7/2, ^-1/2)}$ (as expected).

$\square$