2.5,37 Solve the system of equations and write the solution set as a k-flat:

$$x_1 - 3x_2 + x_3 = 2$$
$$3x_1 - 8x_2 + 2x_3 = 5$$
$$3x_1 - 7x_2 + x_3 = 4.$$

Solution

First, we solve the system of equations by considering the associated augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 2 \\ 3 & -8 & 2 & 5 \\ 3 & -7 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

This corresponds to the equations $x_1 - 2x_3 = -1$, $x_2 - x_3 = -1$, $0 = 0$

or $x_1 = -1 + 2x_3$, $x_2 = -1 + x_3$, $x_3 = x_3$. With $r = x_3$ as a free variable we get: $x_1 = -1 + 2r$, $x_2 = -1 + r$, $x_3 = r$ where $r \in \mathbb{R}$

or $$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1+2r \\ -1+r \\ r \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + r\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \text{ where } r \in \mathbb{R}.$$

with $\vec{a} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ and $W = \operatorname{sp}\left(\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\right)$ we have that solution $\vec{x}$ is in the 1-flat

$$\boxed{\vec{a} + \vec{W} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \operatorname{sp}\left(\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\right).} \quad \square$$