

SECTION 3.1

NUMBER 1

3.1.1. Is the following a vector space:

The set \mathbb{R}^2 , with the usual addition
($[x_1, y_1] + [x_2, y_2] = [x_1 + x_2, y_1 + y_2]$)
and scalar multiplication defined by
 $r[x, y] = [ry, rx]$.

Solution

To show this is NOT a vector space,
we need to violate one of the properties A1-A4
or S1-S4 (see Definition 3.1).

Let's try S4 (I think S3 will do, as well).
S4 says $1\vec{v} = \vec{v}$ for any $\vec{v} \in V$. Notice
 $1[x, y] = [1y, 1x] = [y, x]$. So
we need $[x, y] = [y, x]$. But there
are vectors in \mathbb{R}^2 , say $\vec{v} = [0, 1]$,
where $1[0, 1] = [1 \cdot 1, 0 \cdot 1] = [1, 0] \neq [0, 1]$.
So this set violates S4 and is

NOT a vector space.