

SECTION 3.1
EXERCISE #23

3.1.23 Let V be a vector space. Prove that, if \vec{v} is in V and if r is a scalar and $r\vec{v} = \vec{0}$ then either $r = 0$ or $\vec{v} = \vec{0}$.

Proof

Suppose $r \neq 0$ where $r\vec{v} = \vec{0}$. Then
$$\frac{1}{r}(r\vec{v}) = \frac{1}{r}\vec{0} = \vec{0} \quad \text{by Theorem 3.1(4), "Elementary Properties of Vector Space"}$$
and
$$\frac{1}{r}(r\vec{v}) = \left(\frac{1}{r}r\right)\vec{v} \quad \text{by Definition 3.1 ("Vector Space") Part 53}$$
$$= 1\vec{v}$$
$$= \vec{v} \quad \text{by Definition 3.1 Part 54.}$$

So $\frac{1}{r}(r\vec{v}) = \vec{0} = \vec{v}$ and $\vec{v} = \vec{0}$. That is, either $r = 0$ or (if $r \neq 0$) $\vec{v} = \vec{0}$, as desired. ■