SECTION 3.1

EXERCISE #25

3.1.25 Let $V$ be a vector space, and let $\vec{v}$ and $\vec{w}$ be vectors in $V$. Prove that there is a *unique* vector $\vec{x}$ in $V$ such that $\vec{x}+\vec{v}=\vec{w}$.

*Proof*

First, with $\vec{x}=\vec{w}+(-1)\vec{v}=\vec{w}-\vec{v}$ we have

$\vec{x}+\vec{v}=(\vec{w}-\vec{v})+\vec{v}$

$=\vec{w}+(-\vec{v}+\vec{v})$ by Definition 3.1, "Vector Space", Part A1

$=\vec{w}+(\vec{v}-\vec{v})$ by Definition 3.1 Part A2

$=\vec{w}+\vec{0}$ by Definition 3.1 Part A4

$=\vec{w}$ by Definition 3.1 Part A3,

so $\vec{x}=\vec{w}-\vec{v}$ is *a* solution.

Second, suppose $\vec{y}$ is a solution and $\vec{y}+\vec{v}=\vec{w}$.

Then $(\vec{y}+\vec{v})-\vec{v}=\vec{w}-\vec{v}$

or $\vec{y}+(\vec{v}-\vec{v})=\vec{w}-\vec{v}$ by Definition 3.1 Part A1

or $\vec{y}+\vec{0}=\vec{w}-\vec{v}$ by Definition 3.1 Part A4

or $\vec{y}=\vec{w}-\vec{v}$ by Definition 3.1 Part A3.

So the *only* solution is the solution $\vec{w}-\vec{v}$; that is, there is a unique solution (namely, $\vec{w}-\vec{v}$). ∎