SECTION 3.1

EXERCISE #27

3.1.27 Use the universality of function spaces to explain how we can view the vector space $M_{2,6}$ of 2×6 matrices and the vector space $M_{3,4}$ of 3×4 matrices as essentially the same vector space with just a different notation for the vectors.

<u>Solution</u>

In the last Note of the class notes for this section we see that we can use the set $S = \{(1,1), (2,1), (1,2), (2,2), (1,3), (2,3), (1,4), (2,4), (1,5), (2,5), (1,6), (2,6)\}$ and function $f : S \to \mathbb{R}$ to represent an $m \times n = 2 \times 6$ matrix as

$$M = \begin{bmatrix} f((1,1)) & f((1,2)) & f((1,3)) & f((1,4)) & f((1,5)) & f((1,6)) \\ f((2,1)) & f((2,2)) & f((2,3)) & f((2,4)) & f((2,5)) & f((2,6)) \end{bmatrix}.$$

We can also use function $f : S \to \mathbb{R}$ to represent a matrix in $M_{3 \times 4}$ as

$$N = \begin{bmatrix} f((1,1)) & f((2,2)) & f((1,4)) & f((2,5)) \\ f((2,1)) & f((1,3)) & f((2,4)) & f((1,6)) \\ f((1,2)) & f((2,3)) & f((1,5)) & f((2,6)) \end{bmatrix}.$$

When matrix M is multiplied by a scalar r, the (i,j) entry of rM is $r\,f(i,j)$. When matrix N is multiplied by a scalar r, the entry $f((i,j))$ of N becomes $r\,f((i,j))$ in rN. So scalar multiplication "behaves" in the same way on M and N. If 2×6 matrix M' and 3×4 matrix N' are similarly defined using function $g : S \to \mathbb{R}$ then the (i,j) entry of matrix $M + M'$ is $f((i,j)) + g((i,j))$. The corresponding entry in $N + N'$ is also $f((i,j)) + g((i,j))$. So matrix addition "behaves" the same way as well. The two basic properties of a vector space are scalar multiplication and vector addition. Since these "behave" the same then the vector spaces $M_{2,6}$ and $M_{3,4}$ are essentially the same. $\square$