SECTION 3.1
EXERCISE #29

3.1.29 Referring to the three preceding exercises (particularly Exercise #26), list the vector spaces: $\mathbb{R}^{24}$, $\mathbb{R}^{25}$, $\mathbb{R}^{26}$, $\mathcal{P}_{24}$, $\mathcal{P}_{25}$, $\mathcal{P}_{26}$, $M_{4,7}$, $M_{3,8}$, $M_{3,9}$, $M_{2,12}$, $M_{2,13}$, $M_{4,6}$, and $M_{5,5}$ in two or more columns in such a way that any two vector spaces listed in the same column can be viewed as essentially the same vector space with just different notation for vectors, but two vector spaces that appear in different columns cannot be so viewed. HINT: In Exercise #28 it is shown that $\mathcal{P}_n$ is essentially the same as $\mathbb{R}^{n+1}$.

Solution

From Exercise #26 we have that $M_{m,n}$ is essentially the same as $\mathbb{R}^{mn}$. So representing "essentially the same" as $\cong$ we have: $\mathbb{R}^{24} \cong M_{3,8} \cong M_{2,12} \cong M_{4,6}$, $\mathbb{R}^{25} \cong \mathcal{P}_{24} \cong M_{5,5}$, $\mathbb{R}^{26} \cong \mathcal{P}_{25} \cong M_{2,13}$. Also, $\mathbb{R}^{27} \cong \mathcal{P}_{26} \cong M_{3,9}$. So

$M_{4,7}$	$\mathbb{R}^{24}$	$\mathbb{R}^{25}$	$\mathbb{R}^{26}$	$\mathcal{P}_{26}$
	$M_{3,8}$	$\mathcal{P}_{24}$	$\mathcal{P}_{25}$	$M_{3,9}$
	$M_{2,12}$	$M_{5,5}$	$M_{2,13}$	
	$M_{4,6}$			

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