SECTION 3.2
NUMBER #23

3.2.23 Find a basis for
$sp(1, 4x+3, 3x-4, x^2+2, x-x^2)$ in P.

Solution

By Definition 3.6, we need a linearly independent spanning set for the space. Well the given vectors span ... their span (D'UH), but what about linear independence? So consider the equation

$$r_1(1) + r_2(4x+3) + r_3(3x-4) + r_4(x^2+2) + r_5(x-x^2) = 0(x)$$

OR

$$(r_4 - r_5)x^2 + (4r_2 + 3r_3 + r_5)x + (r_1 + 3r_2 - 4r_3 + 2r_4)1 = 0x^2 + 0x + 0 = 0(x).$$

This requires

$$\begin{aligned} r_4 - r_5 &= 0 \\ 4r_2 + 3r_3 + r_5 &= 0 \\ r_1 + 3r_2 - 4r_3 + 2r_4 &= 0. \end{aligned}$$

The augmented matrix for this system of equations is

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & -4 & 2 & 0 & 0 \end{array}\right] \overset{\text{wd}}{\sim} \left[\begin{array}{ccccc|c} \boxed{1} & 0 & -\frac{25}{4} & 0 & \frac{5}{4} & 0 \\ 0 & \boxed{1} & \frac{3}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \boxed{1} & -1 & 0 \end{array}\right] = H$$

Since a the first, second, and fourth columns of H contain pivots, then a basis is given by the first, second, and fourth vectors: $\boxed{\{1, 4x+3, x^2+2\}}$. □