SECTION 3.2

EXERCISE #45

3.2.45 Use Exercise #41 to describe the solution set of the given differential equation.

(a) $y^{(3)} - 9y' = 0$. HINT! Try to find values of m such that $y = e^{mx}$ is a solution.

Solution

Following the hint, we take $y = e^{mx}$ so that $y' = me^{mx}$, $y'' = m^2 e^{mx}$, and $y^{(3)} = m^3 e^{mx}$.

So $y^{(3)} - 9y' = m^3 e^{mx} - 9me^{mx} = (m^3 - 9m)e^{mx}$

$$= m(m-3)(m+3)e^{mx}$$

and $y^{(3)} - 9y' = 0$ for $m = 0, +3, -3$. So

So $h_1(x) = e^{0x} = 1$, $h_2(x) = e^{3x}$, and $h_3(x) = e^{-3x}$ are solutions to this homogeneous differential equation. Since the differential equation is third order (it involves third order derivatives) then we expect three independent solutions (see the comment in the text after Exercise #42). So the solution set is

$$\boxed{H = \{r_1(1) + r_2 e^{3x} + r_3 e^{-3x} \mid r_1, r_2, r_3 \in \mathbb{R}\}.}$$ □

(b) $y^{(3)} - 9y' = x^2 + 2x$. HINT: A particular solution is

$$p(x) = \frac{-1}{27}x^3 - \frac{1}{9}x^2 - \frac{2}{81}x.$$

Solution

By Exercise #41, every solution is of the form $p(x) + h(x)$ where $h(x) \in H$. That is, the general solution is

$$\boxed{\frac{-1}{27}x^3 - \frac{2}{9}x^2 - \frac{2}{81}x + r_1 + r_2 e^{3x} + r_3 e^{-3x} \text{ where } r_1, r_2, r_3 \in \mathbb{R}.}$$

□