SECTION 3.2
EXERCISE #5

3.2.5 Determine whether the indicated subset is a subspace of the given vector space:
The set S of all functions f in the vector space W of differentiable functions mapping $\mathbb{R}$ into $\mathbb{R}$ such that $f'(2)=0$.

<u>Solution</u>

We apply Theorem 3.2 ("Test for a Subspace") to see if S is a subspace.

First, we test for closure under vector addition. Let $f, g \in S$ (so that $f'(2)=g'(2)=0$).
Then
$$\frac{d}{dx}\left[f(x)+g(x)\right]\Big|_{x=2} = \left(f'(x)+g'(x)\right)\Big|_2$$
$$= f'(2)+g'(2) = 0+0=0,$$
so $f+g \in S$ and S <u>is</u> closed under vector addition.

Second, let $r \in \mathbb{R}$ be a scalar. Then
$$\frac{d}{dx}\left[rf(x)\right]\Big|_{x=2} = rf'(x)\Big|_{x=2} = rf'(2) = r\cdot 0 = 0$$
and so $rf \in S$ and S <u>is</u> closed under scalar multiplication.

So, by Theorem 3.2, $\boxed{S \text{ is a subspace of } W.}$ □